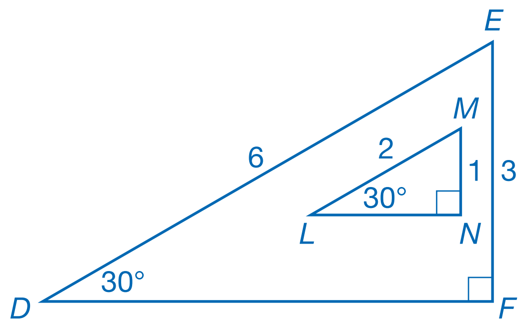
LESSON 12 Trigonometric Ratios

UNDERSTAND Triangles *LMN* and *DEF* both have a 90° angle and a 30° angle, so $\triangle LMN \sim \triangle DEF$ by the AA~ Postulate. Because the triangles are similar, corresponding sides are proportional.

$$\frac{LM}{DE} = \frac{MN}{EF} = \frac{NL}{FD}$$



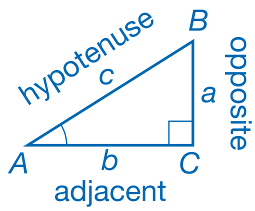
If we take the first two ratios and multiply by $\frac{EF}{LM}$, we get a new proportion: $\frac{EF}{DE} = \frac{MN}{LM}$. The ratio of the **leg** opposite the 30° angle divided by the **hypotenuse** is the same for both triangles. This ratio is called the **sine** ratio.

The ratios of the side lengths of a right triangle depend on the measures of its acute angles. These ratios are called **trigonometric ratios**.

Sine of $\angle A = \sin A = \frac{\text{Opposite leg length}}{\text{Hypotenuse}} = \frac{a}{c}$

Cosine of $\angle A = \cos A = \frac{\text{Adjacent leg length}}{\text{Hypotenuse}} = \frac{b}{c}$

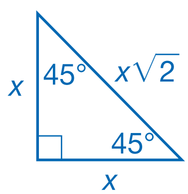
Tangent of $\angle A = \tan A = \frac{\text{Opposite leg length}}{\text{Adjacent leg length}} = \frac{a}{b}$



Thinking "SOH-CAH-TOA" can help you remember these ratios: Sine/Opposite/Hypotenuse-Cosine/Adjacent/Hypotenuse-Tangent/Opposite/Adjacent. Using these ratios, you can find the length of any side of a right triangle if you know one acute angle and any other side.

UNDERSTAND Some right triangles have trigonometric ratios that are easy to remember.

45°–45°–90° triangle:

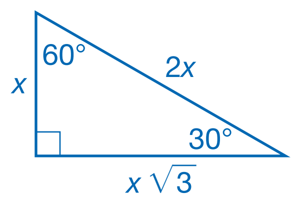


$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

30°–60°–90° triangle:



$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

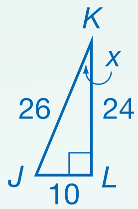
$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

Connect

Consider right triangle JKL. What are the sine, cosine, and tangent ratios for $\angle K$?



1 Identify the opposite and adjacent legs and the hypotenuse.

The leg opposite $\angle K$ is $\overline{JL}$. Its length is 10 units.

The leg adjacent to $\angle K$ is $\overline{KL}$. Its length is 24 units.

The hypotenuse is $\overline{JK}$. Its length is 26 units.

2 Find the trigonometric ratios.

The degree measure of $\angle K$ is x, so use x in the equations.

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{10}{26} = \frac{5}{13}$$

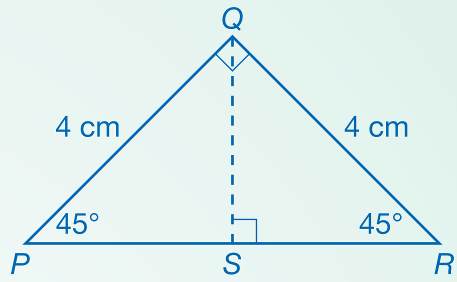
$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{26} = \frac{12}{13}$$

$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{24} = \frac{5}{12}$$

CHECK

The value of x in the triangle is approximately 22.6°. Use this angle measure and a calculator to check that the ratios found above are accurate. For example, press SIN, enter 22.6, and press ENTER. Then verify that the decimal found is about equal to $\frac{5}{13}$ by dividing 5 by 13. Use the COS and TAN keys to check the other ratios.

EXAMPLE A In isosceles right triangle PQR, an altitude has been drawn from angle Q. Use $\overline{QS}$ to find the area of $\triangle PQR$.



1 Find the length of $\overline{PR}$, the hypotenuse of $\triangle PQR$.

Triangle PQR is a $45°-45°-90°$ triangle with legs 4 centimeters long.

The hypotenuse of a $45°-45°-90°$ triangle is $\sqrt{2}$ times the length of a leg, so $PR = 4\sqrt{2}$ cm.

2 Find the length of $\overline{PS}$.

$\overline{QS}$ is an altitude, so $m\angle PSQ = 90°$.

Triangles PSQ and RSQ are both $45°-45°-90°$ triangles with a 4-cm hypotenuse, so they are congruent by ASA Theorem.

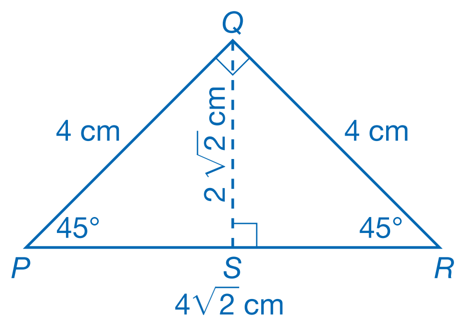
Because congruent parts of congruent triangles are congruent, $PS = RS$ and each segment is half of $\overline{PR}$.

$PS = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

3 Determine the length of $\overline{QS}$.

Because $\triangle PSQ$ is a $45°-45°-90°$ triangle, its legs have equal length.

$QS = PS = 2\sqrt{2}$



4 Calculate the area of $\triangle PQR$.

The area of a triangle is $\frac{1}{2}$ base times height.

$A = \frac{1}{2} \cdot PR \cdot QS$

$A = \frac{1}{2} \cdot 4\sqrt{2} \text{ cm} \cdot 2\sqrt{2} \text{ cm}$

$A = \frac{1}{2} \cdot 8(\sqrt{2})^2$

Recall that if $x^2 = 2$, then $x = \sqrt{2}$. So, substituting $\sqrt{2}$ for x, $(\sqrt{2})^2 = 2$.

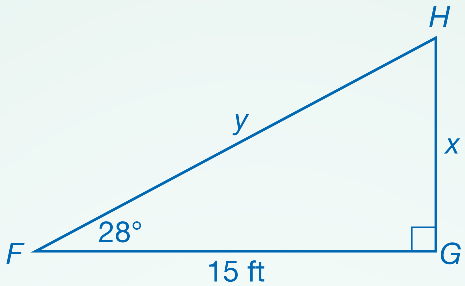
$A = \frac{1}{2} \cdot 8(2)$

▶ $A = 8 \text{ cm}^2$

CHECK

Find the area of $\triangle PQR$, using $\overline{PQ}$ and $\overline{QR}$. Compare the two results.

EXAMPLE B Use trigonometric ratios to find the missing lengths, *x* and *y*, in $\triangle FGH$, to the nearest foot.



1 Calculate the value of *x*.

The given side, which has a length of 15 ft, is adjacent to the given angle, which has a measure of 28°.

The leg labeled *x* is opposite the 28° angle.

The tangent ratio compares the opposite and adjacent sides.

$$\tan 28° = \frac{x}{15}$$

$$(15)(\tan 28°) = x$$

On your calculator, enter 15 × TAN 28 and press ENTER.

▶ $x \approx 8$ ft

2 Calculate the value of *y*.

The leg labeled *y* is the hypotenuse.

The cosine ratio compares the adjacent side and the hypotenuse.

$$\cos 28° = \frac{15}{y}$$

$$(\cos 28°)(y) = 15$$

$$y = \frac{15}{\cos 28°}$$

On your calculator, enter 15 ÷ COS 28 and press ENTER.

▶ $y \approx 17$ ft

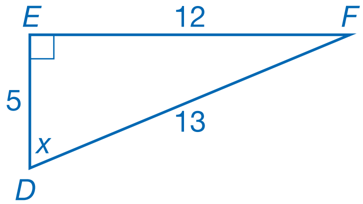
CHECK

Use the Pythagorean Theorem to verify that the side lengths you found for $\triangle FGH$ would form a right triangle.

Practice

Find the sine, cosine, and tangent ratios for $\angle D$ in each triangle.

1.

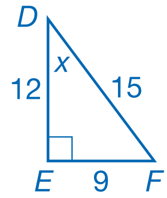


sin x = ________

cos x = ________

tan x = ________

2.



sin x = ________

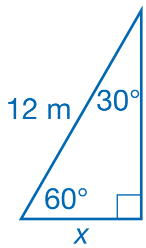
cos x = ________

tan x = ________

REMEMBER SOH-CAH-TOA

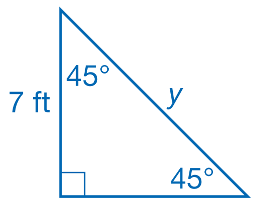
Find the indicated side length in each right triangle.

3.



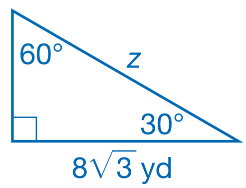
x = ________

4.



y = ________

5.



z = ________

HINT In a $30°-60°-90°$ triangle, the ratio of side lengths is $x : x\sqrt{3} : 2x$.

Fill in the blank with an appropriate word or number.

6. The ________ of an acute angle in a right triangle is the ratio of its adjacent side length to the hypotenuse.

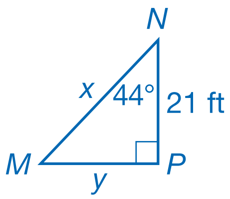
7. If two right triangles are ________, then their corresponding acute angles have identical trigonometric ratios.

8. The hypotenuse of a $45°-45°-90°$ triangle is ________ times longer than each leg.

9. The hypotenuse of a $30°-60°-90°$ triangle is ________ times the length of its shorter leg.

Use trigonometric ratios to find the lengths x and y in each triangle to the nearest foot.

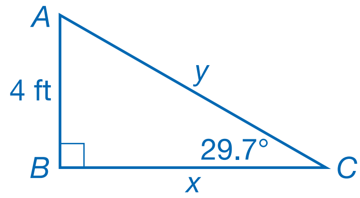
10.



$x \approx$ __________

$y \approx$ __________

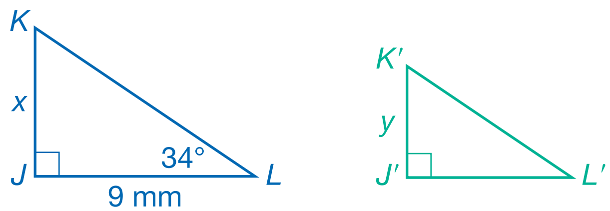
11.



$x \approx$ __________

$y \approx$ __________

Triangle *JKL* was dilated by a scale factor of $\frac{2}{3}$ and translated to the right to form $\triangle J'K'L'$. Use this diagram for questions 12 and 13.



12. How does the tangent of $\angle L$ compare to the tangent of $\angle L'$? Explain.

____

____

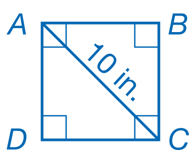
____

13. Find the value of x and the value of y to the nearest millimeter.

____

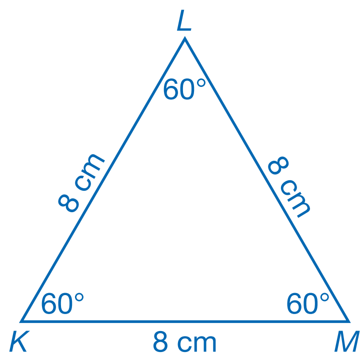
____

14. COMPUTE The diagonal $\overline{AC}$ in the square below is 10 inches long. Compute the exact area of square *ABCD*.



____

15. SHOW Find the exact area of equilateral triangle *KLM*.



____